Direct Camera Calibration from Vanishing Points via Polynomial Solvers

Norio Kosaka @ Workshop on Camera Calibration and Pose Estimation (CALIPOSE) LY Corporation

Available on Zoom for discussion. Apologies for not being on-site.

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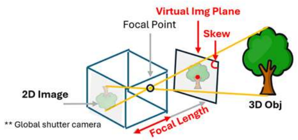
Camera / Vanishing Pts

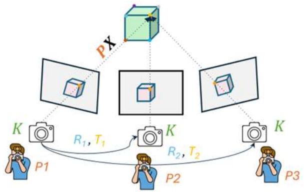
Camera Model

$$K = \begin{bmatrix} f & s & u \\ 0 & g & v \\ 0 & 0 & 1 \end{bmatrix}$$

- Focal length: (f, g)
- Image centre in pixels (u, v)
- Skewness: s



Camera matrix $\mathbf{P} = \mathbf{K}[\mathbf{R}|\mathbf{t}] \in \mathbb{R}^{3\times 4}$



Estimating Params in K is called Camera Calibration

Vanishing Point (VP)

Where parallel world lines appear to intersect in image.

Real-Image *Virtual Image Sphere* [1]

Given 2 orthogonal VPs $u = (u_1, u_2, u_3)^\top$ and $v = (v_1, v_2, v_3)^\top$, linear relation is known to hold [2];

$$u^\top (KK^\top)^{-1} v = u^\top \omega v = 0$$

** ω is called Image of Absolute Conic (IAC).

[1] Lin, Yancong, et al. "Deep vanishing point detection: Geometric priors make dataset variations vanish." CVPR-22.
[2] R. I. Hartley and A. Zisserman. Multiple View Geometry in Computer Vision, 2nd Edition. Cambridge University Press, 2004.

Stratified Approach

Stratified Approach: Solve for ω (Baseline)

- Parameterize IAC.

$$\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \\ \omega_2 & \omega_4 & \omega_5 \\ \omega_3 & \omega_5 & \omega_6 \end{bmatrix} \rightarrow w = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \\ \omega_6 \end{bmatrix}$$

- Stack n measured VPs.
  - $A = \begin{bmatrix} u_1^\top \omega v_1 = 0 \\ \vdots \\ u_n^\top \omega v_n = 0 \end{bmatrix}_{n\times 6} \rightarrow Aw = 0$

Solver: SVD + Matrix Decomposition

- Solve $Aw = 0$ by SVD to find the nullspace (w)
- Recover K by Cholesky decomposition $(\omega = LL^\top)$

Expanded IAC (Ours)

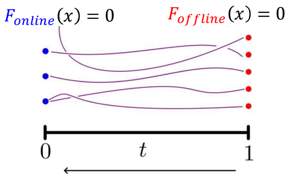
- Use the original algebraic expressions in IAC

$$\omega = \frac{1}{f^2 g^2} \begin{bmatrix} g^2 + s^2 & -s & -ug^2 + svg \\ -s & f^2 & su - f^2 v - s^2 v \\ -ug^2 + svg & su - f^2 v - s^2 v & f^2 g^2 + f^2 v^2 + (gu - sv)^2 \end{bmatrix}$$

- Stack n measured VPs as before;
  - $A = \begin{bmatrix} u_1^\top \omega v_1 = 0 \\ \vdots \\ u_n^\top \omega v_n = 0 \end{bmatrix} \rightarrow Aw = 0$ $\quad n \times 6$

Solve via Polynomial Solver

- Use the *continuation method* to solve the resulting poly-sys
- *Hot-start*; Start from a system in the family of the target system;
  - $F_{offline}$ approximates F_{online} but uses parameters computed offline.



Algebraic Analysis

Variations of calibration tasks

- Example task: $ffuv0$
  - Fixed focal length $(\frac{f}{g} = 1)$ + Zero Pixel skew $(s = 0)$

$$K = \begin{bmatrix} f & 0 & u \\ 0 & f & v \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \omega = \begin{bmatrix} \omega_1 & 0 & \omega_2 \\ 0 & \omega_1 & \omega_3 \\ \omega_2 & \omega_3 & \omega_4 \end{bmatrix}$$

- Similarly, there could be various assumptions on K!

Setup

- *Data* (Coefficients of Polynomial System)
  - Synthetically generated n orthogonal VPs
- *Metrics:* computed in *Macaulay2*
  - dim(I): Dimension of Solution-set
  - deg(I): Degree of Polynomial System

Toy example: $I = \langle y - x^2 \rangle \subset \mathbb{C}[x, y]$
- Dim=1 as it's a curve
- Degree=2 as intersects w/h a generic line at 2 pts

Results

- Simpler tasks (e.g., $1100s$)
  - Minimal problems (dim = 0)
- Complex tasks (e.g., $fguvs$)
  - Underconstrained despite being square.
  - Likely due to algebraic dependencies.

	Pattern	dim(I)	deg(I)
0-Dimension	1100s	0.0 ± 0.0	1.0 ± 0.0
	11uvs	0.0 ± 0.0	1.0 ± 0.0
	f1000	0.0 ± 0.0	1.0 ± 0.0
Positive Dimension	ffuv0	1.0 ± 0.0	1.0 ± 0.0
	fgu00	1.0 ± 0.0	1.0 ± 0.0
	fguvs	1.0 ± 0.0	1.0 ± 0.0

Solver Evaluation

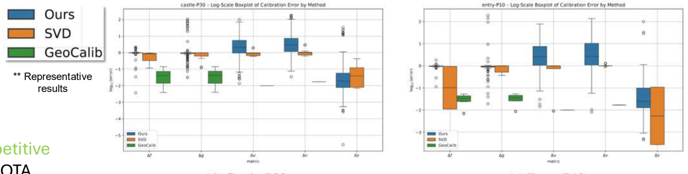
Synthetic

- *Data* (Coefficients of Polynomial System)
  - Synthetically generated n orthogonal VPs
- *Comparisons*
  - PHC-HS (Direct): Ours
  - PHC-HS (Stratified): Stratified w/h PHC
  - SVD: Stratified approach using SVD
- *Metrics*: MSE in intrinsics (fguvs)
- *Results*
  - Solver Succ. Rate: Ours (100%) vs SVD (40%).
  - Accuracy: Ours achieved the lowest errors.

Metric	PHC-HS (Direct) (Ours!!)	PHC-HS (Stratified)	SVD
Task: fguvs			
Δf	5.64e−14 ± 1.77e−13	0.467 ± 0.338	6.15e−14 ± 7.42e−14
Δg	6.28e−14 ± 1.81e−13	0.583 ± 0.709	8.08e−14 ± 1.28e−13
Δu	1.73e−13 ± 5.62e−13	0.151 ± 0.107	8.63e−5 ± 2.56e−4
Δv	1.83e−13 ± 4.91e−13	0.149 ± 0.109	1.97e−13 ± 3.04e−13
Δs	3.17e−14 ± 9.39e−14	0.0106 ± 0.0083	5.89e−5 ± 1.71e−4
Task: ffuv0			
Δf	7.26e−14 ± 2.52e−13	0.629 ± 0.621	4.39e−14 ± 4.36e−14
Δu	1.63e−13 ± 6.37e−13	0.134 ± 0.087	9.35e−14 ± 9.16e−14
Δv	2.67e−13 ± 8.96e−13	0.118 ± 0.087	9.04e−14 ± 7.61e−14

Real Img

- *Data*: Strecha et al. 2008
- *Comparisons*
  - PHC-HS (Direct): Ours
  - SVD: Stratified approach using SVD
  - GeoCalib: Learning-based SOTA
- *Results*
  - PHC-HS (Direct): 100% SR and competitive errors compared to learning-based SOTA



(d) Castle-P30 (e) Entry-P10